Stability and Robustness of Dynamical Traffic Networks

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Outline of the Talk

- Motivation
- From static to dynamical traffic networks
  - Dynamics = capacity constraints + route choice + traffic control
- Stability and resilience
- Conclusion and future work
Motivation

- Costs of traffic congestion [TTI TAMU urban mobility report 2012]
  - Financial cost: $121 Billion
  - Time wastage: 5.5 Billion hours
  - Health, environment, etc.
Motivation

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- Vulnerability to arbitrary and malicious ‘shocks’

Typical Monday at 18:30
**Motivation**

- **Costs of traffic congestion** [TTI TAMU urban mobility report 2012]
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  - Health, environment, etc.

- **Vulnerability to arbitrary and malicious 'shocks'**

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  Monday 11/07/11 at 18:30
Renewed interest: limited capacity of physical infrastructure, rapid advancements in information technology.

Cyber physical: wireless devices as sensory and actuation modes.

Transportation networks prone to disruptions.

Why distributed?

- Increased resilience to failure of control modules
- Scalability with respect to network size
- On-board computation
- Trade-off between performance and distributedness
Key elements of traffic models

- Infrastructure capacity
- Traffic light
- Driver choice
- Congestion pricing
Flow capacity on every link
Flow conservation at every node
Maximum feasible load = bottle-neck capacity
Network flow

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- Flow conservation at every node
- Maximum feasible load = bottle-neck capacity

Static framework
- Centralized
Network flow

- Flow capacity on every link
- Flow conservation at every node
- Maximum feasible load = bottle-neck capacity

Framework of choice for planning purposes
Traffic distribution is the outcome of a non-cooperative game between drivers.

- Driver decisions are dynamic.
- Driver decisions are myopic.
Congestion games

Traffic distribution is the outcome of a non-cooperative game between drivers.
Driver decisions are dynamic.
Driver decisions are myopic.

Equilibrium outcome
Adaptability to disturbances
Static
Global decision dynamics
From static to dynamical model

- $f_{j \rightarrow e}$ flow routed from $j$ to $e$

- $f_{e}^{\text{out}} = \sum_{\text{outgoing } j} f_{e \rightarrow j}$

- $f_{e}^{\text{in}} = \sum_{\text{incoming } j} f_{j \rightarrow e}$
Stability and resilience of transportation networks
Quantifying stability and resilience

Stability

- Network is stable if output equals input
- For unstable networks, delay is infinite
- Response to ‘small’ disturbances

$\lambda_{in} \rightarrow \lambda_{out}(t)$
Quantifying stability and resilience

### Stability
- Network is stable if output equals input
- For unstable networks, delay is infinite
- Response to 'small' disturbances

\[ \lambda_{\text{in}}(t) \rightarrow \lambda_{\text{out}}(t) \]

### Resilience
- Link disturbance = loss in capacity
- Network disturbance = $\sum$ link disturbances
- Smallest malicious disturbance that destabilizes the network
Influence of route choice decisions

\[ f_{e \rightarrow j} = D_e G_j \]

\( G_j \): fraction of drivers choosing link \( j \)

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**Cooperative route choice decisions**

- business as usual congestion
  \[ \implies \text{business as usual decision} \]

\[ G_j^*(\text{eqm}) = \text{eqm route choice} \]

- choose links with less congestion

\[ \frac{\partial G_j^*}{\partial \rho_k} \geq 0 \]

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Example: i-logit

utility \( i \) = myopia + inertia
Cooperative route choice decisions

- business as usual congestion
  \[ \implies \] business as usual decision
- choose links with less congestion

Example: i-logit
- utility \( _i = \text{myopia} + \text{inertia} \)

If the load on the system is feasible, then \( G^* \) is stabilizing
- Within the constraint of not controlling the inflow, \( G^* \) performs best
- \( G^* \) does not give the maximum possible resilience
- The gap increases with the network size
Cooperative route choice decisions

- business as usual congestion $\Rightarrow$ business as usual decision
- choose links with less congestion

Example: i-logit
- utility $i = \text{myopia} + \text{inertia}$

- If the load on the system is feasible, then $G^*$ is stabilizing
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Resilience $= \min \text{ node residual capacity}$
Examples of suboptimal route choice

- passive routing
Examples of suboptimal route choice

- passive routing

- aggressive routing
Back to the general case

\[ \dot{\rho}_e = f_{e}^{in} - f_{e}^{out} \]

- \( f_{j \rightarrow e} \) flow routed from \( j \) to \( e \)

\[ f_{e}^{out} = \sum_{\text{outgoing } j} f_{e \rightarrow j} \]

\[ f_{e}^{in} = \sum_{\text{incoming } j} f_{j \rightarrow e} \]
Cooperative routing

Boundary conditions

- Empty link $\implies$ no outflow
- No flow towards congested links
- Fully congested links give maximum outflow if there is room downstream
Cooperative routing

- Increase in congestion $\implies$ increase in outflow
  \[
  \frac{\partial f_{j\rightarrow e}}{\partial \rho_j} \geq 0
  \]

- Avoid congested links
  \[
  \frac{\partial f_{j\rightarrow e}}{\partial \rho_k} \geq 0
  \]

- Increase in downstream congestion $\implies$ decrease in outflow
  \[
  \frac{\partial f_{\text{out}}}{\partial \rho_k} \leq 0
  \]
Cooperative routing

- Increase in congestion $\implies$ increase in outflow
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- Avoid congested links
  \[ \frac{\partial f_{j \rightarrow e}}{\partial \rho_k} \geq 0 \]

- Increase in downstream congestion $\implies$ decrease in outflow
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- Control based on local information
- Backward propagation of information
Performance of cooperative routing

- Feasible load $\implies$ network is stable
- Infeasible load $\implies$ there exists a unique bottleneck which gets jammed simultaneously.
- Entire network is shut down or no link is jammed
Performance of cooperative routing

- Feasible load $\Rightarrow$ network is stable
- Infeasible load $\Rightarrow$ there exists a unique bottleneck which gets jammed simultaneously.
- Entire network is shut down or no link is jammed

Maximum possible network stability and resilience
Resilience = network residual capacity
Graceful failure
Implications for planning

- Quantitative framework for resilience

- Dependence of resilience on traffic load, network structure, link capacity and route choice behavior

- Resilience as a social objective for transportation planning

- Resilience not aligned with typical social objectives such as delay
Current and future work

- Comprehensive study of resilience under a variety of practical constraints on traffic flow
- From analysis to control of traffic flow
- Connection between agent-based and macroscopic models
- Tradeoff between resilience and delay
- Extension to other infrastructure networks
Traffic flow theory

Cell Transmission Model for Networks:

- Outflow from link $e$ depends on congestion on $j$ and $k$
- Ratio between $f_{e \rightarrow j}$ and $f_{e \rightarrow k}$ is independent of congestion on $j$ and $k$
From static to dynamical model

Mass conservation

\[ \dot{\rho}_e = f_{e}^{\text{in}} - f_{e}^{\text{out}} \]

Constraints

- Density capacity on every link
- Flow capacity on every link

\[ f_{e}^{\text{in}} \text{ and } f_{e}^{\text{out}} \text{ depend on traffic flow, route choice and signal control} \]

dynamic